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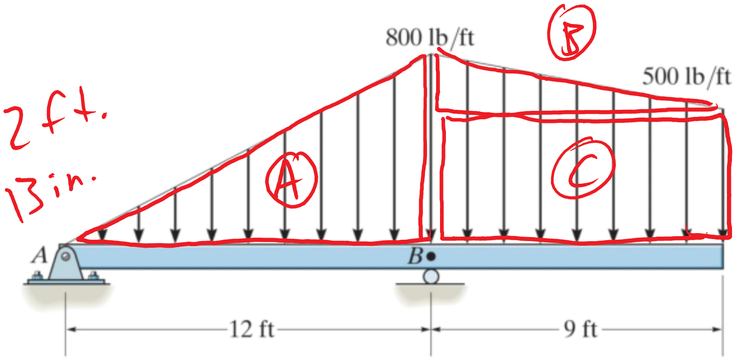
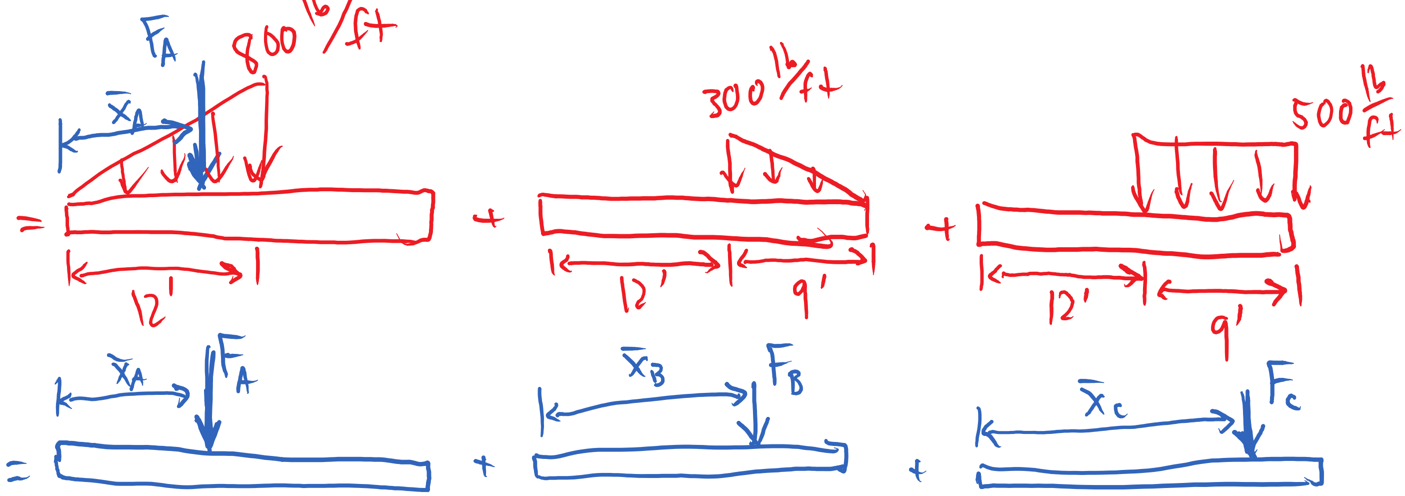
Replace the loading by an equivalent resultant force and specify its location on the beam measured from point B

$12' \equiv 12$ ft.

$13'' \equiv 13$ in.





$\bar{x}_A = 8'$

$F_A = \frac{1}{2}(12')(800\ \frac{lb}{ft})$

$= 4800$ lbs

$\bar{x}_B = 12' + \frac{1}{3}(9') = 15'$

$F_B = \frac{1}{2}(9')(300\ \frac{lb}{ft})$

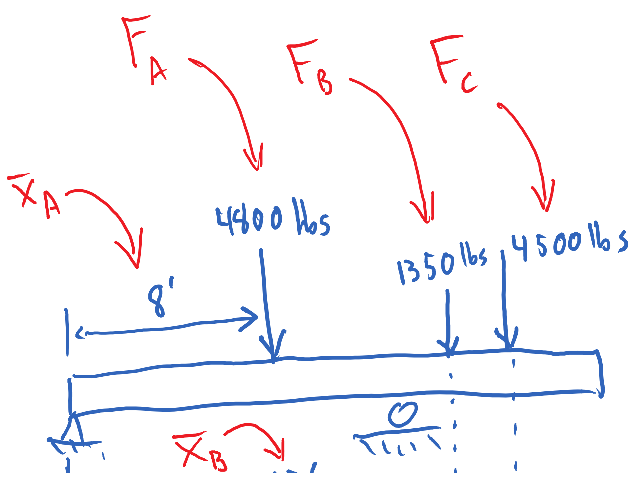
$= 1350$ lbs

$\bar{x}_c = 12' + \frac{9'}{2}$

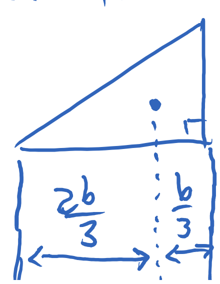
$\bar{x}_c = 16.5'$

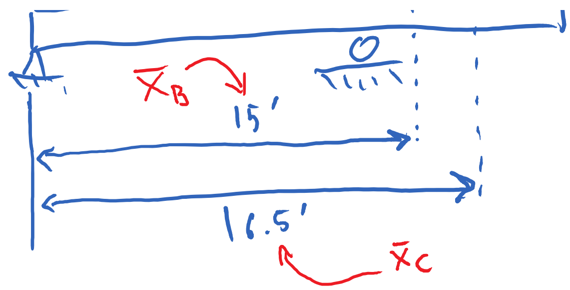
$F_c = (500\ \frac{lb}{ft})(9')$

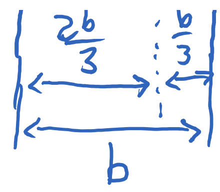
$= 4500$ lbs



centroid of a triangle







Now, $F_R = \sum F = 4800\,\text{lbs} + 4500\,\text{lbs} + 1350\,\text{lbs}$

$= 10650\,\text{lbs}$

Where does F_R act?

$$\left.\bar{x}_A \cdot F_A + \bar{x}_B \cdot F_B + \bar{x}_C \cdot F_C = \bar{x}_R \cdot F_R \right\}\text{equivalent moment}$$

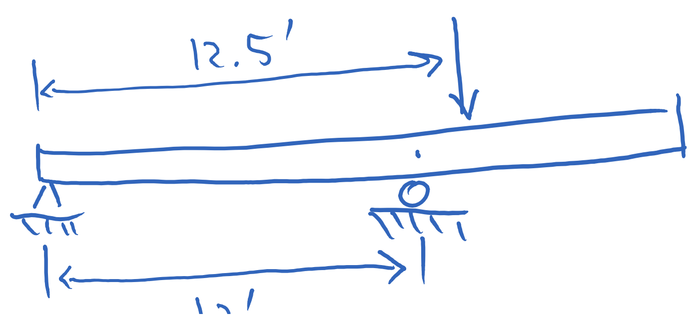
$$\bar{x}_R = \frac{\bar{x}_A \cdot F_A + \bar{x}_B \cdot F_B + \bar{x}_C \cdot F_C}{F_R}$$

$$= \frac{(8')(4800\,\text{lbs}) + (15')(1350\,\text{lbs}) + (16.5')(4500\,\text{lbs})}{10650\,\text{lbs}}$$

$\bar{x}_R = 12.5\text{ ft}$

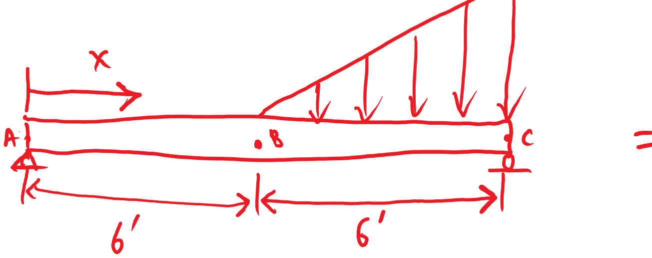
From point B

$F_R = 10650\,\text{lbs}$



12'

$\Rightarrow$ F_R acts 0.5 ft to the right of the roller support.



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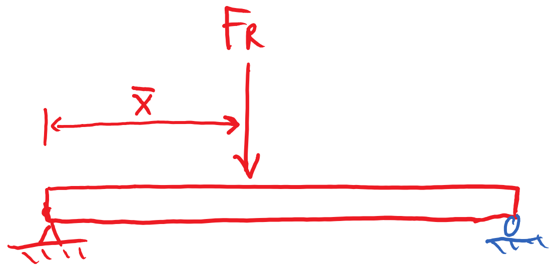


$F_R = $?

A) 600 lbs
B) 300 lbs
C) 1200 lbs
D) 900 lbs
E) None of Above

$\bar{x} = $?

A) 6'
B) 12'
C) 8'
D) 10'
E) None of Above

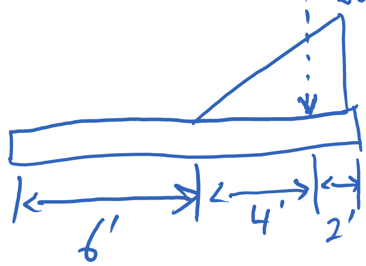
$$F_R = \frac{1}{2} \cdot b \cdot h$$

b: 6'

h: 100 lb/ft

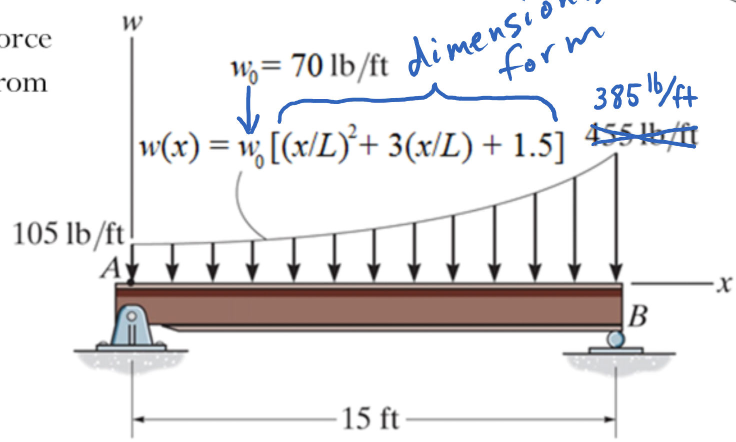
$$= \frac{1}{2}(6')(100 \text{ lb/ft}) = 300 \text{ lbs}$$

F_R acts $\frac{1}{3}$ of the base of the triangle from the tall end of the triangle



$$\Rightarrow \bar{x} = 6' + 4' = 10'$$

Replace the loading by an equivalent resultant force and specify its location on the beam measured from point A



$$F = \int_0^L \underbrace{w(x)}_{\text{force/length}} \cdot \underbrace{dx}_{\text{length}}$$

$$= w_0 \cdot \int_0^L \left[\left(\frac{x}{L}\right)^2 + 3\left(\frac{x}{L}\right) + 1.5\right] \cdot dx \cdot \frac{L}{L}$$

$0 < x < 15'$

$L = 15'$

$\Rightarrow 0 < \frac{x}{L} < 1$

$$= w_0 \cdot L \cdot \int_0^1 \left[\left(\frac{x}{L}\right)^2 + 3\left(\frac{x}{L}\right) + 1.5\right] d\left(\frac{x}{L}\right)$$

let $a = \frac{x}{L}$

$$= w_0 \cdot L \int_0^1 a^2 + 3a + 1.5 \cdot da$$

$$= w_0 \cdot L \cdot \left(\frac{1}{3}a^3 + \frac{3}{2}a^2 + 1.5a\right)\Big|_0^1$$

$$= w_0 \cdot L \cdot \left(\frac{1}{3} + \frac{3}{2} + \frac{3}{2}\right)$$

$$= \frac{10 w_0 \cdot L}{3}$$

$$= \frac{10}{3}\left(70 \frac{\text{lbs}}{\text{ft}}\right)(15\text{ ft})$$

$$= 3500 \text{ lbs}$$

$M = \bar{x}_R \cdot F_R$

$\Rightarrow \bar{x}_R = \frac{M}{F_R}$

Numerator gives
moment, M

$= 3500 \text{ lbs}$

the moment, M

$$\bar{x} = \frac{\int_0^L w(x) \cdot x \cdot dx}{\int_0^L w(x) \cdot dx} = \frac{\int_0^L w(x) \cdot x \cdot dx}{3500 \text{ lbs}}$$

$$\int_0^L w_0 \cdot \left[\left(\frac{x}{L}\right)^2 + 3\left(\frac{x}{L}\right) + 1.5\right] \cdot x \cdot \underline{dx} \quad \times \frac{L^2}{L^2}$$

$$= w_0 \cdot L^2 \cdot \int_0^1 \left[\left(\frac{x}{L}\right)^2 + 3\left(\frac{x}{L}\right) + 1.5\right] \cdot \frac{x}{L} \cdot \underline{d\left(\frac{x}{L}\right)}$$

$$= w_0 \cdot L^2 \cdot \int_0^1 \left(a^3 + 3a^2 + 1.5 \cdot a\right) \cdot da$$

$$= w_0 \cdot L^2 \cdot \left(\frac{a^4}{4} + a^3 + \frac{3}{4}a^2\right)\Big|_0^1$$

$$= w_0 \cdot L^2 \cdot \left(\frac{1}{4} + 1 + \frac{3}{4}\right)$$

$$= 2 \cdot w_0 \cdot L^2$$

$$= 2\left(70 \frac{\text{lbs}}{\text{ft}}\right)(15 \text{ ft})^2$$

$$= \left(140 \frac{\text{lbs}}{\text{ft}}\right)(225 \text{ ft}^2)$$

$$= 31{,}500 \text{ lb-ft}$$

$$\boxed{\bar{x} = \frac{31500 \text{ lb-ft}}{3500 \text{ lb}} = 9 \text{ ft}}$$

[Inclass_Lecture_Chapter5_EquilibriumRigidBodies.pptx]

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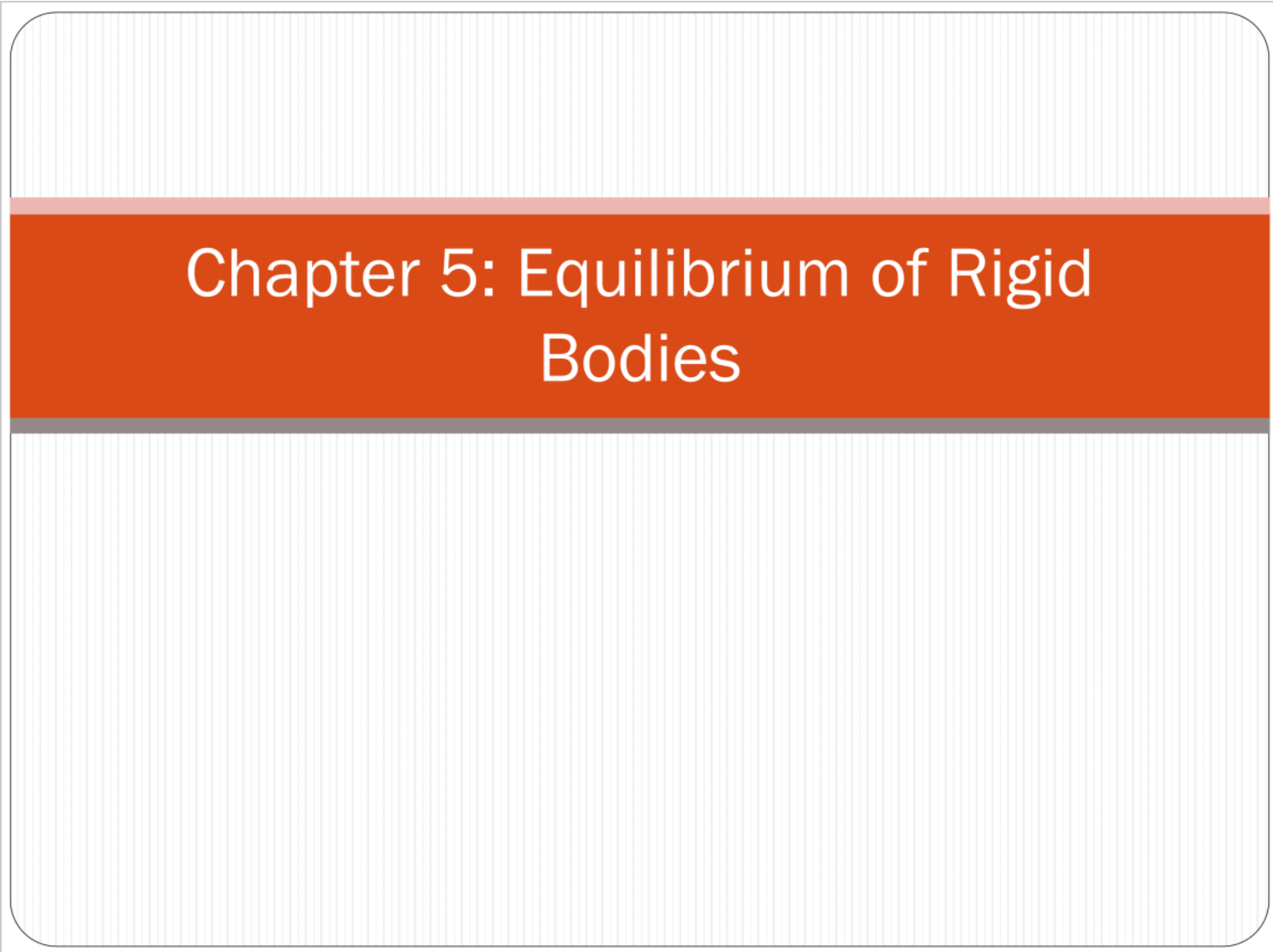

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Equilibrium of a Rigid Body

Static equilibrium:

$\sum \boldsymbol{F} = \mathbf{0}$ (zero forces = no translation)

$\sum(\boldsymbol{M}) = \mathbf{0}$ (zero moment = no rotation)

Maintained by reaction forces and moments

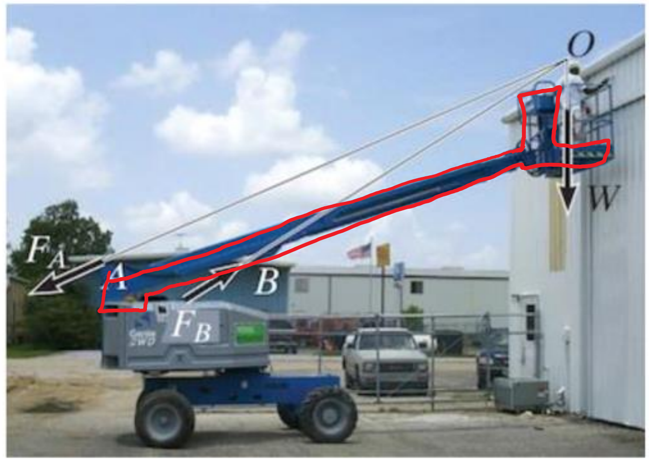
forces from supports / constraints are exactly enough to produce zero forces and moments

Assumption of rigid body

Shape and dimensions of body remain **unchanged** by application of forces.

More precisely:

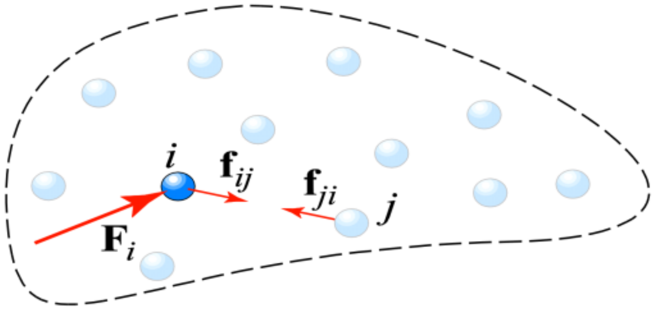
All **deformations of bodies** are small enough to be ignored in analysis.



Equilibrium of a Rigid Body

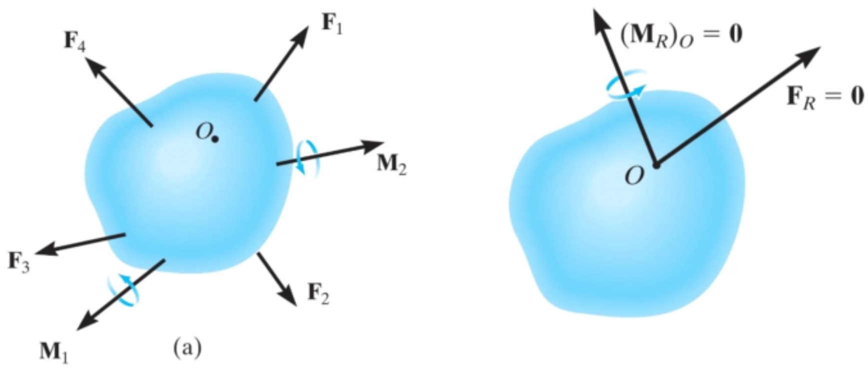
This subject is of central importance in statics. We regard a rigid body as a collection of particles.

$\boldsymbol{F}_i$ = resultant external force on particle i
$\boldsymbol{f}_{ij}$ = internal force on particle i by particle j
$\boldsymbol{f}_{ji}$ = internal force on particle j by particle i



Note that $\boldsymbol{f}_{ji} = \boldsymbol{f}_{ij}$ by Newton's third law and therefore the internal forces will not appear in the equilibrium equations.

We can reduce the force and couple moment system acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point O.



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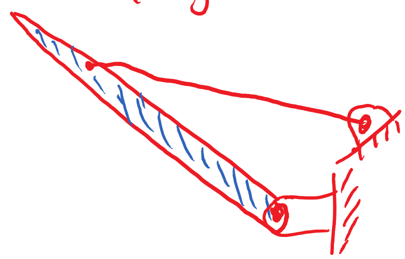
Process of solving rigid body equilibrium problems

1. Create idealized model (modeling and assumptions)

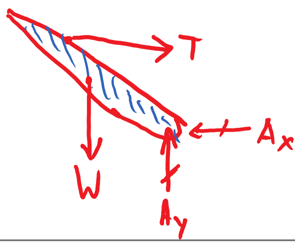
System

Subsystem (rigid body)



2. Draw free body diagram showing ALL the external (applied loads and supports)

Free Body Diagram



3. Apply equations of equilibrium

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M = 0$$

usually take ΣM about a support